CHARGED DROPLETS EVAPORATION AND MOTION IN AN ELECTRIC FIELD

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Abstract—Monodisperse spray evaporation is investigated theoretically when a pure liquid or an electrolyte solution spray is charged and moves through an electric field. The solution of the equations in the case of electrolyte solutions gives the droplet size evolution down to the "equilibrium radius" when the relative humidity is high and down to the saline kernel when the humidity is lower. This solution also gives the dynamic behaviour in an electric field when the droplets are charged and are moving in a gas stream.

A non dimensional curve is obtained for a given humidity, molality and temperature. independently of the electric field. With this curve it is possible to predict the droplet evolution only knowing a "middle time" of evaporation, calculated for a given electric force and a given initial radius.

1. INTRODUCTION

Spray behaviour can be generally described by a spray equation (Williams 1965) analogous to Boltzmann's equation in the kinetic theory of gases, and by the various conservation relations. Only simplified solutions have been obtained for this system of equations, using Maxwell's (1890) assumption.

Spalding (1959) obtained analytic solutions for the monodisperse spray combustion; he took into account through Frössling's (1938) factor the increase of the evaporation owing to the droplets' velocity relative to the gas, and expressed the drag coefficient including the effect of the outward mass-transfer at the droplet surface. Recently, Law (1975) has adapted Spalding's model to the case of evaporation within an adiabatic flow and also to the case of evaporation within an isothermic flow, or combustion.

The above investigation applies to spray of a pure liquid. The studies concerning the droplets of a mixture are less extensive. El Golli (1975) has studied theoretically and experimentally the behaviour of the droplets of electrolyte solutions floating freely within a turbulent gas stream. The present paper makes it possible to extend Law's model for a charged spray moving in an electric field. This spray can be constituted by a pure liquid or by an electrolyte solution. In the latter case the action of an electric field causes droplets to develop a velocity relative to the gas stream. For a constant electric force, that relative velocity increases as the evaporation occurs. The evaporation in turn is accelerated as the velocity increases. The droplets' motion and the evolution of their size are therefore closely interrelated.

Such conditions can exist, for instance, in electrostatic spray guns and in some energy conversion electrohydrodynamic devices. In particular, such conditions exist in a "static" operated wind generator, where direct wind energy conversion into electrical energy would be obtained by wind carrying charged water droplets.

2. GOVERNING EQUATIONS

The two-phase flow, involving a spray and its vapor, in an inert gas is assumed to be steady and one dimensional. Throughout the flow the temperature (T_g) and the density (ρ_g) of the inert gas remain constant. Temperature (T_c) and density (ρ_c) of the liquid change during evaporation in the case of an electrolyte solution. In the case of pure water, ρ_c does not vary and, as assumed by Law (1975), T_c remains constant. The macroscopic properties are considered uniform in each cross section of the flow. In addition the spray is assumed to be monodispersed and sufficiently scattered for the droplets' interaction to be negligible.

The two phase behaviour can be so described by the mass-transfer and energy equations which yield the evaporation rate equation, by the mass conservation equation, and by the equation of equilibrium of forces. We denote u_{c_0} the initial droplet velocity and u_{g_0} the gas stream velocity; ϵ_c and ϵ_g respectively represent the condensed and gaseous mass flux fractions.

2.1 Droplet evaporation rate

The droplet evaporation rate equation can be established by the following assumptions: the evaporation is quasi-steady for spherical droplets in unbounded gaseous and isobaric atmosphere; the various transfer coefficients and specific heats are constant; the Lewis number is unity.

In such conditions, with the equations of concentration and temperature gradients, Law (1975) obtained an expression for the evaporation rate equation for a radius r_c, at time t:

$$\frac{d\omega}{d\tilde{t}} = -\frac{3}{2}(1 - \omega)^{1/3} \frac{\ln(1 + B)}{\ln(1 + B_0)} F(Re, Sc),$$ [1]

where

$$\omega = 1 - r_c^3/r_{c_0}^3$$ [2]

represents the mass fraction that is vaporized,

$$\tilde{t} = 1 - t \frac{2\rho_g D}{\rho_c r_{c_0}^2} \ln(1 + B_0)$$ [3]

is a non-dimensional time variable, D being the diffusion coefficient,

$$B = \frac{(C_p)_g}{L_c}(T_g - T_c)$$ [4]

is the transfer number, L_c being the specific heat of vaporization and $(C_p)_g$ the gas specific heat,

$$F(Re, Sc) = 1 + ZU^{1/2}(1 - \omega)^{1/6}$$ [5]

is the Frössling's factor in which:

$$Z = 0.276Re_0^{1/2}Sc^{1/3},$$ [6]

with

$$Re_0 = \frac{2\rho_g u_{g_0} r_{c_0}}{\mu_g(1 - \epsilon_c)},$$ [7]

and

$$Sc = \frac{\mu_g}{\rho_g D},$$ [8]

(μ_g is the gas viscosity),

$$U = \bar{u}_g - \bar{u}_c$$ [9]

with

$$\bar{u}_g = (1 - \epsilon_c)u_g/u_{g0} \text{ and } \bar{u}_c = (1 - \epsilon_c)u_c/u_{g0}.$$

2.2 Mass conservation

The overall mass conservation given by Williams (1965) can be written in a non-dimensional manner:

$$\bar{u}_g = 1 - \epsilon_c(1 - \omega). \tag{10}$$

2.3 Droplet equilibrium equation within the flow

2.3.1 Drag law. The drag force (T) acting on a droplet moving in a gas stream with a relative velocity $u_g - u_c$, and a mass m_c is:

$$T = \frac{3}{8}\frac{\rho_g}{\rho_c} m_c \frac{(u_g - u_c)^2}{r_c} C_D; \tag{11}$$

the drag coefficient C_D, given by Spalding (1959) is:

$$C_D = [Re^{-1}K(Sc)F(Re, Sc)]G(B), \tag{12}$$

with $K(Sc)$ a function only depending on the constant parameter Sc and $G(B)$ expressed by:

$$G(B) = \frac{\ln(1 + B)}{B}. \tag{13}$$

Details of these coefficients have been given by Law (1975). Taking into account the preceding relations, [11] can be written in non-dimensional form:

$$\frac{dU}{d\bar{t}} = \frac{d\bar{u}_g}{d\bar{t}} + \frac{3}{2}[SU(1 - \omega)^{-2/3}(G(B)/G(B_0))F(Re, Sc)], \tag{14}$$

where

$$S = \frac{G(B_0)}{\ln(1 + B_0)}\frac{ScK(Sc)}{16}. \tag{15}$$

2.3.2 Electric force. For an electric field strength E, the electric force which acts on a droplet with a charge Q, is:

$$F = Q(r_c)E(x). \tag{16}$$

The charge Q is assumed to be a constant during the evaporation.

2.3.3 Equilibrium of directly applied forces to a charged droplet. When a charged droplet moves in a gaseous stream and through an electric field, a drag force and an electric force act on it. The equation expressing the equilibrium of the forces is:

$$T - Q(r_c)E(x) - m_c\gamma = 0, \tag{17}$$

where γ is the droplet acceleration.

An easy calculation shows that, in the cases with which we are concerned, the term $(-m_c\gamma)$ representing the force of inertia is negligible during the whole of its trajectory within the electric field.

Consequently, the preceding equation becomes:

$$T - Q(r_c)E(x) = 0. \tag{18}$$

Taking into account the drag law, we find

$$Q(\omega)E(\bar{x})W = SUF(Re, Sc)(1 - \omega)^{1/3}\frac{B_0}{B}\ln(1 + B) \qquad [19]$$

with

$$W = \frac{1 - \epsilon_c}{4\pi r_{c_0}u_{g_0}\rho_g D}. \qquad [20]$$

2.4 Spray evolution

With the governing equations [1], [10] and [19], it is possible to describe the spray evolution by resolving the system. For that, we eliminate the term $F(Re, Sc)$ dividing the two members of the diffusion equation [1] by the corresponding members of [19]:

$$\frac{d\omega}{d\bar{t}} = -\frac{3}{2}\frac{Q(\omega)E(\bar{x})W}{SU\ln(1 + B_0)}\frac{B}{B_0}. \qquad [21]$$

Considering that the initial mass flux of the condensed phase is sufficiently small, the mass conservation equation is reduced to

$$\bar{u}_g = 1. \qquad [22]$$

Finally, [19] can be written as a cubic polynomial of $\sqrt{U}$, ω being a parameter

$$(\sqrt{U})^3\left[(1 - \omega)^{1/2}SZ\frac{B_0}{B}\ln(1 + B)\right] + (\sqrt{U})^2\left[(1 - \omega)^{1/3}\frac{B_0}{B}\ln(1 + B)\right] - WQ(\omega)E(\bar{x}) = 0. \qquad [23]$$

Equations [21]–[23] form the new simplified system which is resolved as follows. Knowing the droplet initial radius, the time step Δt is chosen and [23] provides the value of $U(\omega)$. Once $U(\omega)$ is determined, droplet radius variation can be found during the chosen time step, and consequently the radius values at time $t + \Delta t$. Step by step, all the droplet evolution can be so found in as much as B can be determined at every moment.

2.5 Transfer number variation

The value of the transfer number B, at any given moment, results from the temperature difference between the droplet and the surrounding gas. Assuming a quasi-steady evaporation, it is possible to express B directly starting from the relation which exists between the heat and the mass fluxes during the vaporization. Moreover, assuming that the partial vapor pressure at the surface of the droplet is equal to the saturating pressure of the liquid at temperature T_c, one obtains:

$$T_g - T_c = \frac{L_c D}{\lambda_g}(C_c - C_g) \qquad [24]$$

where λ_g is the thermal conductivity of the gas, C_c the saturated vapor concentration at the droplet surface, and C_g the vapor concentration value at infinity.

The value of C_g is to be determined from the gas temperature and the relative humidity. The values of C_c and T_c can then be calculated by use of [24] and tables giving the values of C_c as function of temperature T_c.

In the case of electrolyte solutions the saturated vapor concentration is given by the product of the solvant activity a_L by the saturated vapor concentration of the pure solvent at the same temperature. Consequently, in [24] C_c must be replaced by $a_L C_c$.

Taking [4] into account, the transfer number can be written:

$$B = \frac{(C_p)_g D}{\lambda_g}(a_L C_c - C_g).$$ [25]

Robinson & Stokes (1959) provide tables giving activity in terms of solution concentration.

3. RESULTS

3.1 General remarks

The cases of pure water and of sodium chloride solution's droplets in an air stream at atmospheric pressure have been investigated by computing the preceding equations. We have considered the case of an uniform electric field and of a constant droplet charge. The electric field remains uniform in so far as the spray creates a weak enough space charge. The droplets charge can be considered constant as long as the field at the surface of the droplet does not exceed the disruptive field in the atmosphere, and does not create an electrostatic pressure in excess of the inner pressure due to the surface tension, Whitby & Liu (1966).

We have been led therefore to choose initial charges so that those limits are not exceeded when the droplet radius reaches: the equilibrium radius in the case of electrolyte solutions; a critical radius, at which stage the calculation is stopped, in the case of pure water.

Qualitatively, the whole of the results confirm what could be anticipated, i.e. that, all other factors being equal, the vaporization is all the more rapid as: the electric force, and consequently the relative velocity, is greater, the temperature is higher; the relative humidity is lower; and the electrolyte concentration is lower.

Contrary to what happens in the case of pure water, the vaporization is not always complete in the case of an electrolyte solution.

For high relative humidity, the droplet reaches a limit concentration value such as the saturated vapor pressure of the solution becomes equal to the partial water vapor pressure in the atmosphere; evaporation then stops and the droplet radius remains constant thereafter.

On the contrary, for low relative humidity, when the solution becomes saturated in electrolyte, the solution vapor is still higher than the pressure of water vapor in the atmosphere and the evaporation continues.

Two events can then occur: (a) The solute crystallizes. The solution concentration remains constant and equal to the saturation concentration, and evaporation continues until a pure salt kernel is obtained. (b) There is no crystallization. The solution becomes more and more super-saturated; its saturated vapor pressure decreases and, when it reaches the water vapor pressure in the atmosphere, the evaporation stops.

Establishing the equations, the case of crystallization has not been considered. That is why, the part of the curves corresponding to supersaturated solution will be represented in dotted line.

After a few examples illustrating the incidence of the various parameters, we shall see that the evolution of the droplets can be predicted in an easy way through the use of non-dimensional curves and a "middle time" which concept will be defined later (cf. §3.6).

3.2 Electric force influence

Figure 1 shows the droplet radius variations against time, for various values of product QE, when the gas stream temperature is 25°C and the relative humidity is 0.8, the spray being constituted with droplers whose initial radius is equal to 100 μm; salt concentration is 20 g/kg of water. Curves 1–4 correspond to electric forces respectively equal to $0, 2 \cdot 10^{-11}, 2 \cdot 10^{-8}$ and
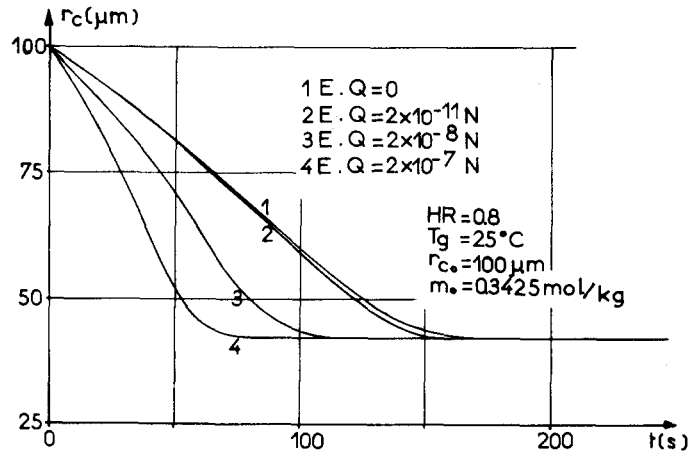
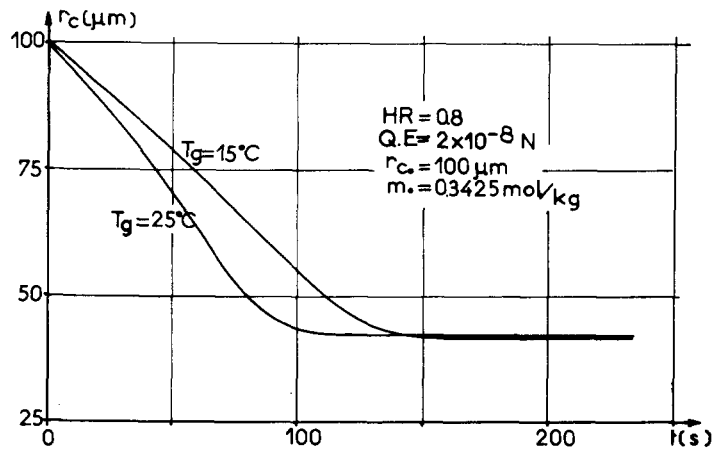Figure 1. Radius evolution for different values of the electric force.

Figure 2. Radius evolution for two different values of the gas temperature.

$2 \cdot 10^{-7}$ N. Note that $2 \cdot 10^{-8}$ N represents the electric force which acts on a droplet being in $5 \cdot 10^5$ V/m field and carrying a $4 \cdot 10^{-14}$ C charge, in other words $2.5 \cdot 10^5$ elementary charges.

In the particular case of a droplet with a zero relative velocity in the gas stream ($QE = 0$) one finds the same results as the one given by El Golli's equations (1975).

3.3 Influence of the ambient temperature

Figure 2 shows the gas stream temperature influence upon droplets evaporation for conditions identical to those of curve 3 in figure 1.

Evaporation is slower if the temperature is lower, but the droplet radii at which it ceases are never appreciably different from one another, that means that evaporation ceases for practically equal concentrations. This can be explained by the fact that the two curves are given for the same relative humidity so that when the temperature changes, the partial water vapor pressure in the atmosphere varies in the same proportion as the saturating vapor pressure of the solution. However the equilibrium radius is slightly smaller, and consequently, the equilibrium concentration is slightly higher at 15°C than at 25°C. This results from the fact that the solvent activity decreases slightly as temperature increases.

3.4 Relative humidity influence

Figures 3 and 4 show the relative humidity influence upon the evolution of the droplets
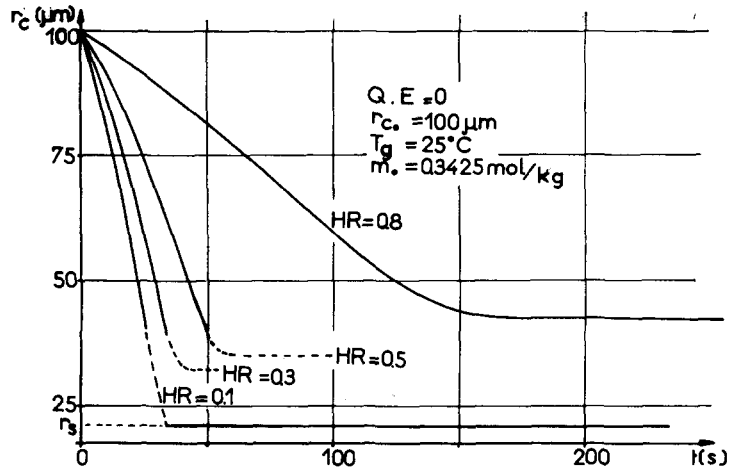
Figure 3. Radius evolution for different relative humidity (without electric force).
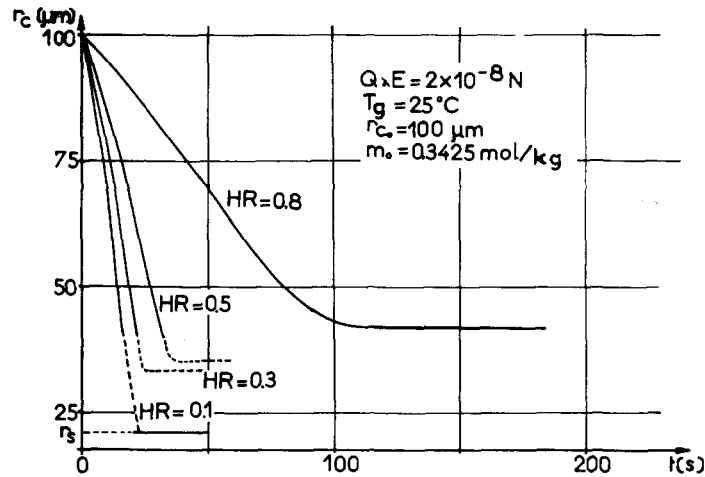

Figure 4. Radius evolution for different relative humidity (with an electric force).

respectively in the case of no charge or no field and of a $2 \cdot 10^{-8}$ N electric force. In both cases the evaporation time is much reduced when relative humidity decreases.

For relative humidity greater than 76%, the equilibrium radius corresponds to a solution whose molality is lower than saturation which is about 6 mol/kg for the sodium chloride. For relative humidity smaller than 76% (see dotted part of the curves), the computation supplies equilibrium radius corresponding either to supersaturated solutions, or to a salt kernel.

In fact, if the droplet crystallizes, evaporation will continue in all cases down to the same limit which is the salt kernel with an "equivalent radius" r_s.

3.5 Concentration influence

Figure 5 gives an example of concentration influence upon droplet radius variations against time. Contrary to the case of pure water ($m_0 = 0$), an equilibrium radius is obtained for the solutions even with low initial modality. The equilibrium radius is already equal to nearly half of the initial radius for a solution only containing 10 g sodium chloride in 1 l of water.

Increasing the initial concentration causes an important decrease of the evaporation rate. However, let us note that these figures relate to a case when relative humidity is 80%. For lower humidities, the differences with pure water are less appreciable.
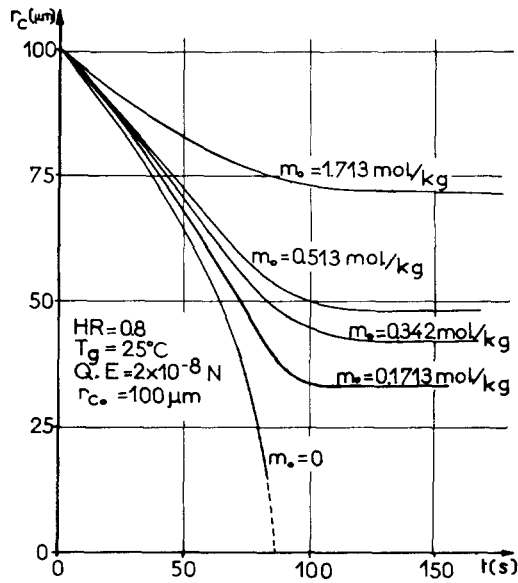
Figure 5. Radius evolution for different values of the molality.

3.6 Non-dimensional curves

All the results obtained concerning the radius variation of the droplet against time can be assembled to evolve non-dimensional curves, by drawing, for various molalities (m_0) the representative curves

$$\frac{r_c}{r_{c_0}} = f\left(\frac{t}{t_m}\right),$$

where t_m is the "middle time" necessary for the droplet to reach a radius r_c equal to $1/2(r_{c_0} + r_l)$, r_{c_0} representing the initial radius and r_l the equilibrium radius.

These curves are independent of the acting electric force QE. Their shape changes very slightly when the initial radius changes, and in practice, the curves of figure 6 are valid for droplets the initial radius of which may reach several hundred μm.
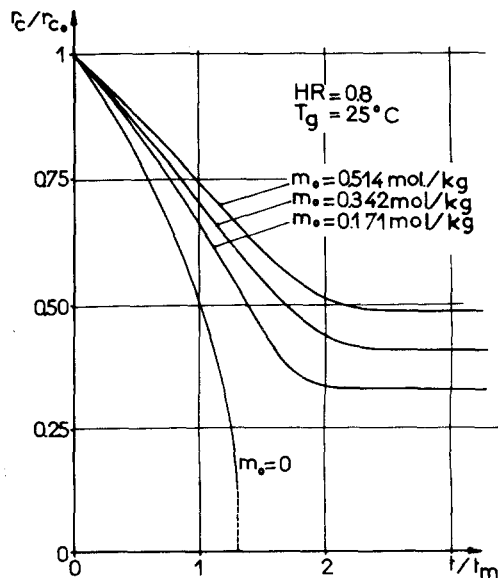

Figure 6. Radius evolution: non-dimensional curves.

To use these curves it is necessary to know the value of t_m in every instance. Figures 7–9 give those values in terms of electric force QE respectively for initial droplet radius of 1, 10 and 100 μm, and that for relative humidity equal to 80% and a temperature of the surrounding atmosphere of 25°C.

It must be noted that time scales differ from one figure to another and therefore, when all conditions are otherwise equal, the "middle time" of vaporization varies approximately as the inverse square of the initial droplets radius, whenever the electric force is small.

3.7 Droplet motion

For a constant electric force the droplet velocity ($u_r = u_g - u_c$) relative to the surrounding atmosphere increases during evaporation, see figure 10. Depending on the direction and the velocity of the gas stream, the electric force accelerates or brakes the droplet motion referred to fixed axis.

If the droplets are sufficiently decelerated, the movement of the droplets may reverse itself during the vaporization, that inversion taking place when $|u_r| = |u_g|$.

For example, with the conditions of curve $m_0 = 0.342$ mol/kg of figure 10, the inversion of the movement of the droplet would occur after 1.5 sec for a velocity of the main flow equal to 1.3 m/sec. Figure 11 gives the distance travelled by the droplet, measured from the axis carried
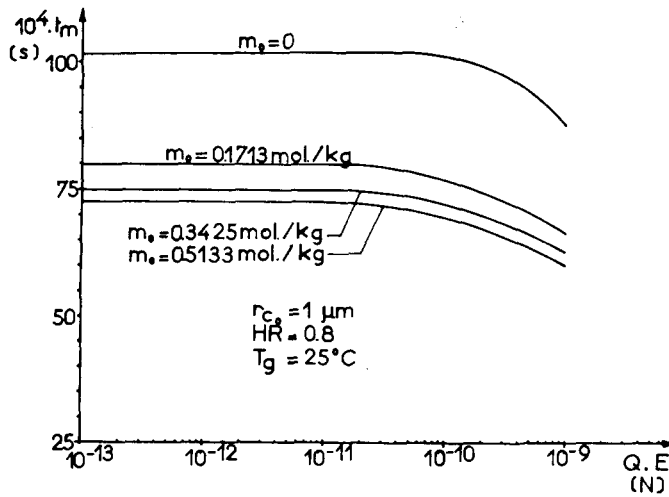

Figure 7. Variation of the "middle time" (initial radius 1 μm).
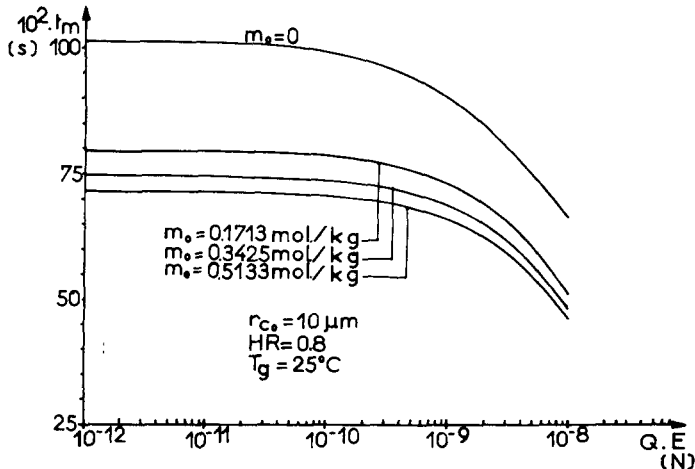

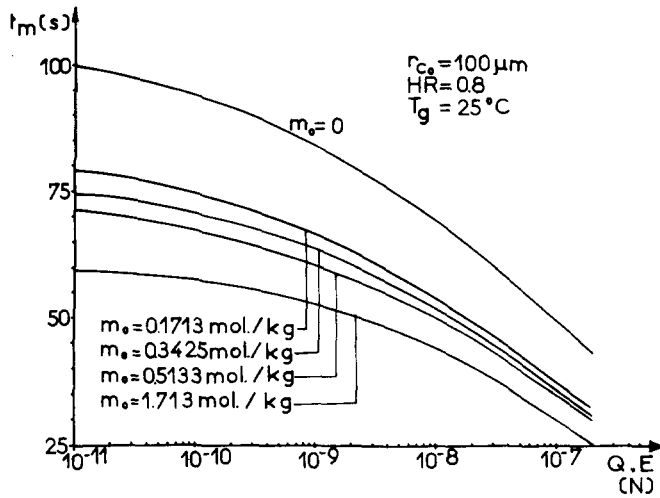Figure 8. Variation of the "middle time" (initial radius 10 μm).

Figure 9. Variation of the "middle time" (initial radius 100 μm).
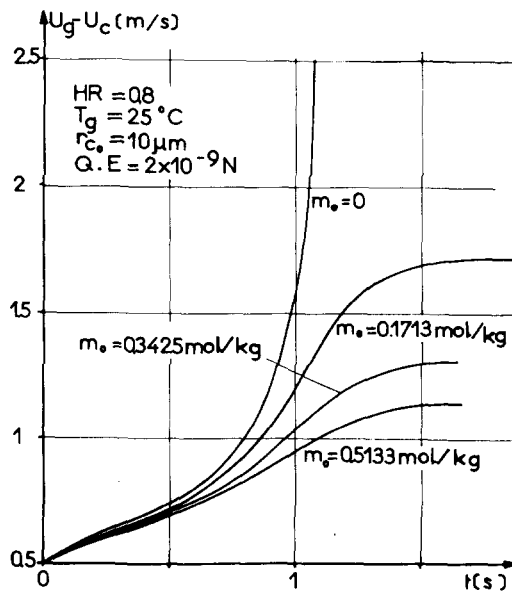

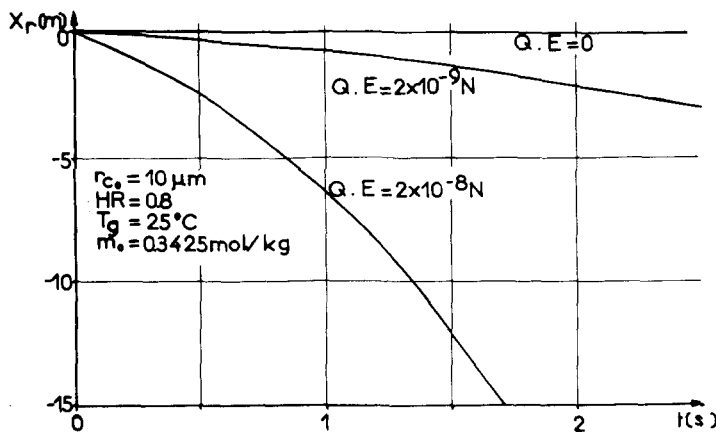Figure 10. Variation of the relative velocity.

Figure 11. Distance travelled by a droplet, measured from axis carried by the flow.

by the flow, for various values of the electric force. In the given example the inversion occurs at −1.35 m, which means a travelled distance of 0.6 m referred to a fixed axis.

4. CONCLUSION

The present method of computation enables the prediction of the evolution of pure water or electrolyte solution spray, electrically charged and submitted to an electric field. It can be used in so far as the charge of the droplet remains constant during evaporation, i.e. that there are neither corona discharge, nor droplet burst, as long as there is no crystallization in the case of electrolyte solutions. However these last phenomena generally occur toward the end of the evaporation only.

The electric field creates a relative droplet velocity referred to the gaseous stream, and thereby the evaporation is accelerated. The resolution of the equations enables the determination of the droplets evolution and motion as function of time, for different values of electric force, air relative humidity and salt concentration in the liquid.

The same equations can be applied to the case of sprays constituted with various liquids and gas phases.

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